Institute for Geometry and its Applications
School of Mathematical Sciences
University of Adelaide

Workshop on Several Complex Variables and Complex Geometry

11–12 August 2007

PROGRAM

The workshop will be held on the University of Adelaide's North Terrace campus in room G08 of the Mathematics Building.

Saturday 11 August

9:30: Coffee (tea room, 2nd floor)

10:00-10:50: Jens Kroeske, University of Adelaide
Invariant differential pairings

11:00-11:50: Rasul Shafikov, University of Western Ontario
Real-analytic sets in complex spaces and CR maps

Lunch

14:00-14:50: Alexander Isaev, Australian National University
Proper group actions in complex geometry

Coffee

15:30-16:20: Nicholas Buchdahl, University of Adelaide
A new proof of a theorem of Kodaira

16:30-17:20: Finnur Lárusson, University of Adelaide
The Siciak-Zahariuta extremal function and analytic discs

Sunday 12 August

9:00-9:50: Gerd Schmalz, University of New England
Dynamical behaviour of CR automorphisms near a fixed point

Coffee

10:30-11:20: Rasul Shafikov, University of Western Ontario
Holomorphic extension of CR functions from quadratic cones

11:30-12:20: Michael Eastwood, University of Adelaide
The X-ray transform on complex projective space

ABSTRACTS

Nicholas Buchdahl, University of Adelaide
A new proof of a theorem of Kodaira

In 1963, Kodaira proved that every compact Kähler surface is a deformation of an algebraic surface. It was long conjectured that the same might be true in higher dimensions—that is, that every compact Kähler manifold is a deformation of an algebraic manifold. However, in 2003 Voisin gave examples in every dimension greater than three of compact Kähler manifolds which could not be diffeomorphic to any algebraic manifold. Efforts in attempting to understand the underlying phenomena in dimensions three and above have led to a short and relatively easy proof of Kodaira's original theorem, and it is that proof which will be presented in this talk.

Michael Eastwood, University of Adelaide
The X-ray transform on complex projective space

The classical Radon transform takes a function on the plane and integrates it over the straight lines in the plane. Its invertibility provides the mathematical basis of modern medical imaging techniques. The X-ray transform takes a function in three-space and integrates it over the straight lines, the terminology being motivated by medical imaging. As one might expect, both of these transforms are best viewed on real projective space. In this talk, I shall discuss what happens on complex projective space where the straight lines are the Fubini-Study geodesics. This is joint work with Hubert Goldschmidt.

Alexander Isaev, Australian National University
Proper group actions in complex geometry

In their celebrated paper of 1939 Myers and Steenrod showed that the group of isometries of a Riemannian manifold acts properly on the manifold. This fact has many important consequences. In particular, it implies that the group of isometries is a Lie group in the compact-open topology. This result triggered extensive studies of closed subgroups of the isometry groups of Riemannian manifolds. The peak of activities in this area occurred in the 1950's–70's, with many outstanding mathematicians involved: Kobayashi, Nagano, Yano, H.-C. Wang, Egorov, to name a few. In particular, Riemannian manifolds whose isometry groups possess subgroups of sufficiently high dimensions were explicitly determined.

I will speak about proper actions in the complex-geometric setting. In this setting (real) Lie groups act properly by holomorphic transformations on complex manifolds. My general aim is to build a theory parallel to the theory that exists in the Riemannian case. In my lecture I will survey recent classification results for complex manifolds that admit proper actions of high-dimensional groups.

Jens Kroeske, University of Adelaide
Invariant differential pairings

It is generally known (see [**?**]), that on an arbitrary manifold M one can write down the Lie derivative $\mathcal{L}_X\omega_b$ of a one-form $\omega_b \in \Omega^1(M)$ with respect to a vector field $X^a \in TM$ in terms of an arbitrary torsion-free connection ∇_a as

$$\mathcal{L}_X\omega_b = X^a\nabla_a\omega_b + \omega_a\nabla_b X^a,$$

where the indices used are abstract in the sense of [**?**]. This **pairing** is obviously linear in X^a and in ω_b, i.e. **bilinear**, **first order** and **invariant** in the sense that it does not depend upon a specific choice of connection. One can specify an equivalence class of connections and ask for invariance under change of connection within this equivalence class. In conformal geometry, for example, one deals with an equivalence class of connections that consists of the Levi-Civita connections that correspond to metrics in the conformal class.

In this talk we will deal with complex manfiolds M with an AHS structure and those manifolds come equipped with an equivalence class of connections (see [**?**]). Moreover every representation $(\mathbb{V}, \rho)$ of the structure group P (where G/P is the (flat) model space for M, G being a complex semi-simple Lie group and P a maximal parabolic subgroup) gives rise to an associated bundle V and we will examine M-th order invariant bilinear differential pairings between sections of associated bundles

$$\Gamma(V) \times \Gamma(W) \longrightarrow \Gamma(E).$$

It turns out that first order bilinear invariant differential pairings on the homogeneous models G/P can be classified (apart from certain totally degenerate cases) and that this construction yields invariant pairings even in the general (curved) case.

Higher order pairings on homogeneous model spaces can be constructed with methods similar to those used in the classification of linear invariant differential operators (see [**?**]). In the general curved case these methods fail. However, one can try to simulate the *liftings* that are required in the construction using tractor calculus. Some results in this direction will be presented and problems addressed.

References

[1] A. Cap, J.Slovák, and V. Soucek, *Invariant operators on manifolds with almost Hermitian symmetric structures, 3. Standard operators*, Diff. Geom. Appl. **12** (2005) 51–84.

[2] M.G. Eastwood and J.W. Rice, *Conformally invariant differential operators on Minkowski space and their curved analogues*, Commun. Math. Phys. **109** (1987) 207–228. Erratum, Commun. Math. Phys. **144** (1992) 213.

[3] R. Penrose and W. Rindler, *Spinors and Space-time, Volume 1*, Cambridge University Press 1984.

Finnur Lárusson, University of Adelaide

The Siciak-Zahariuta extremal function and analytic discs

The polynomial hull $\hat{K}$ of a compact set K in $\mathbb{C}^n$ is the set of all z such that $|P(z)| \leq \sup_K |P|$ for all complex polynomials P. If z lies in an analytic disc with boundary in K, then it easily follows from the maximum principle that $z \in \hat{K}$. A famous example of Stolzenberg (1963) shows that the converse fails. The question of whether polynomial hulls could still be somehow described in terms of analytic discs remained open until 1993, when Poletsky gave a description using his theory of disc functionals. In 2006, Ragnar Sigurdsson and I obtained a different description as an application of our work, published in 2005, on the so-called Siciak-Zahariuta extremal function. The gist of both results is to suitably relax the boundary conditions on the analytic discs. I will describe my work with Ragnar and its background.

Gerd Schmalz, University of New England

Dynamical behaviour of CR automorphisms near a fixed point

The presence of an automorphism is a strong condition on a CR manifold. In the strictly pseudoconvex case automorphisms whose iterates push a given point arbitrarily close to a fixed point can only exist if the CR manifold is locally equivalent to a sphere. This can be considered as a CR version of the Wong-Rosay theorem. In the case of an embedded nonquadratic manifold with indefinite Levi form we show that an automorphism that is non-attracting in the normal direction must be hyperbolic in CR direction. In this case only light-like points belonging to the manifold itself can be pushed to the fixed point by iterates of the automorphisms. If the automorphism is attracting in the normal direction elliptic, parabolic or hyperbolic behaviour may occur. This is joint work with K. T. Kim.

Rasul Shafikov, University of Western Ontario

Real-analytic sets in complex spaces and CR maps

I will discuss a particular question concerning real analytic sets defined in complex spaces. As an application it is proved that a continuous map from a real analytic CR manifold whose graph is a real analytic set, and which is CR on a non-empty open set, is CR.

Holomorphic extension of CR functions from quadratic cones

In joint work with D. Chakrabarti, we consider one-sided holomorphic extension of CR functions from a special class of real analytic hypersurfaces in the spirit of Trepreau.